Propensity Score Stratification

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Description of Example

- Objective: Estimate the treatment effect of having one full-time security personnel in schools on the proportion of harsh discipline measures administered.

- Treated: schools with one full-time security employee (n1 = 521)
- Untreated: schools without any full-time security staff (n0 = 1,265)

- Outcome: harsh disciplinary practices e defined as removing students from the school with no additional services for the remainder of the year, transferring to specialized schools, or suspending for at least 5 days.

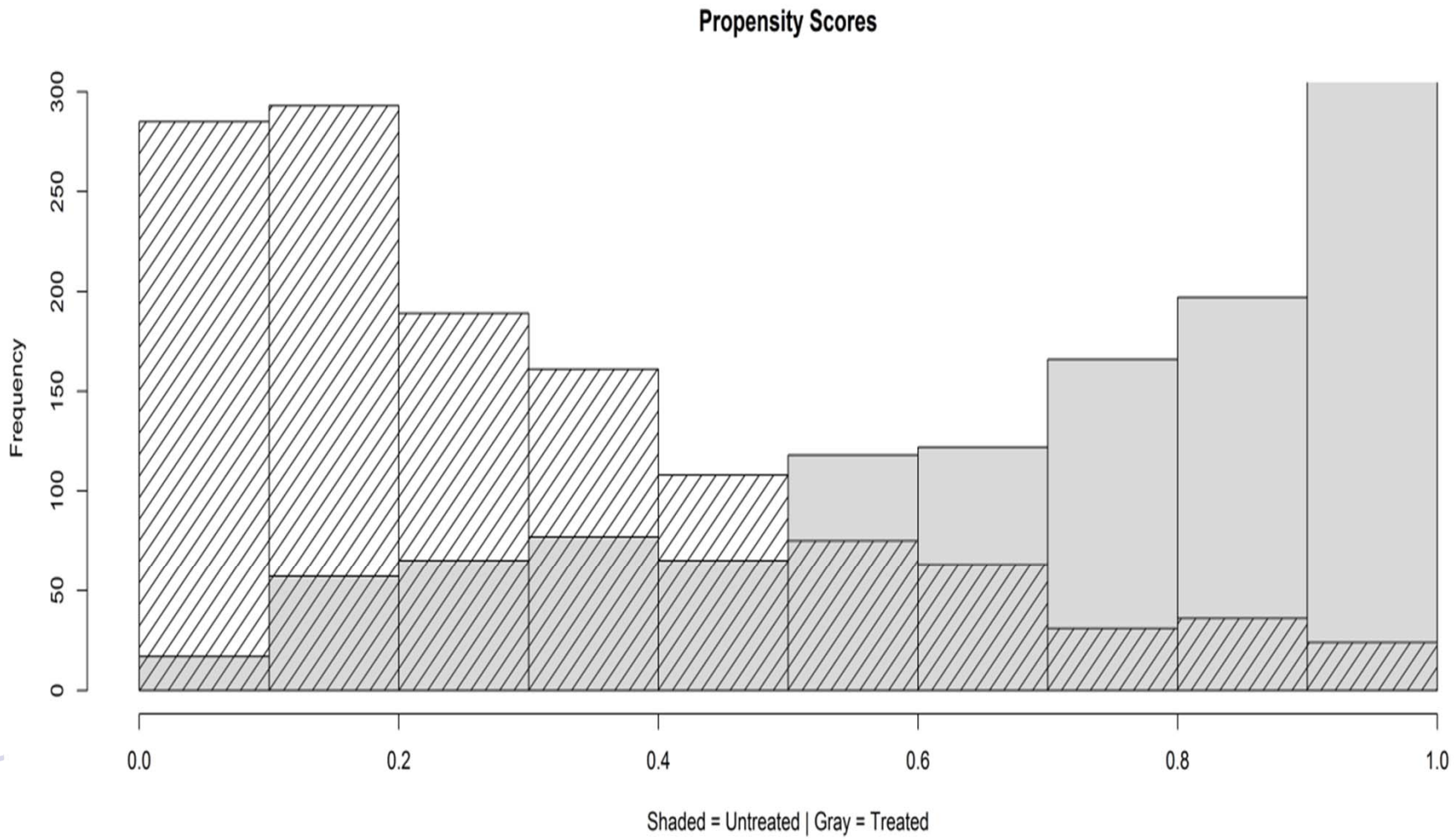
- Dataset: Public use data from the 2007–2008 wave of the SSOCS[2]

Propensity Scores for Example

- Propensity scores for this example were estimated with logistic regression. Missing data was handled with single imputation.

- The covariates were 67 variables including percentages of students in special education, eligible for free or reduced-price lunch, and minority students, average daily attendance, crime level in the school's area, school locale, and grade level), indicators of the security measures used in the school (e.g., use of locks, uniforms, training of teachers, written plans).

Common support for example



Theoretical Justification of Propensity Score Stratification

- If the sample is divided into strata such that all units within a stratum have the same e(x) and at least one unit in the stratum is exposed to each condition, the expected difference in treatment means within each stratum is the ATE at that e(x), and the weighted average of the stratum differences is an unbiased estimated of the ATE.

Options to stratify propensity scores

1. Use quantiles of the entire sample, resulting in approximately the same stratum sizes.

```
 subclass
treat   1   2   3   4   5
    0 340 303 276 209 137
    1  18  54  81 148 220
```

1. Use quantiles of the treated, resulting in approximately equal treated sizes per stratum.

```
 subclass
treat         1   2   3   4   5
  Untreated 796 223 124  72  50
  Treated   104 104 105 103 105
```

Propensity Score Stratification and Common Support Evaluation

- With stratification, common support is adequate if none of the strata have zero or very few treated or untreated;

- Common support can be improved by decreasing the number of strata, but this corresponds to a decrease in removal of bias.

- Removing units at the extremes of the propensity score distribution before stratifying can improve common support, but decreases generalizability of results.

Approaches to propensity score stratification

- 1) Pool strata-specific treatment effects;

- 2) Obtain a marginal treatment effect across strata (marginal mean weighting through stratification).

Propensity Score Stratification with pooling of treatment effects

- Step 1: Use the PS to stratify all observations into $j = 1, \ldots, k$ strata
  - Most frequently quintiles are used, which have been shown to remove approx. 90% of the bias, given that SI holds.
  - More strata remove more bias but there needs to be at least one treated and one control observation per stratum.
- Step 2: Evaluate covariate balance with strata
- Step 3: Calculate observation weights
- Step 4:Obtain the effect within each stratum then average the effects weighted by strata proportions

Step 2: Covariate balance evaluation within strata

- Summary of standardized mean differences on 67 covariates for each stratum:

	Stratum				
	1	2	3	4	5
Minimum	–0.16787	–0.28865	–0.34637	–0.4921	–0.42214
Mean	0.005179	0.001641	–0.03004	–0.02564	–0.06004
Maximum	0.25173	0.264709	0.39388	0.25578	0.37623

- In this example, covariate balance was not adequate within any stratum.

Options when covariate balance is not achieved within strata

- Increate the number of strata;
- Change the form of the propensity score model or the method to estimate propensity scores;
- Remove within-stratum residual bias by combining stratification with weighting, matching, or direct covariate adjustment with analysis of covariance, or regression estimation of group means within strata;
- Use marginal mean weighting through stratification

Step 3: Stratum Weights:

$$ATE: \quad w_k = n_k / n$$

$$ATT: \quad w_k = n_{1k} / n_1, \text{ where } k = \text{stratum}$$

Stratum	$k = 1$	$k = 2$	…	$k = K$	sum	
Z = 0	n_{01}	n_{02}	…	n_{0k}	n_0	
Z = 1	n_{11}	n_{12}	…	n_{1k}	n_1	← ATT
sum	$n_{.1}$	$n_{.2}$	…	$n_{.k}$	n	← ATE

Step 3: Stratum Weights

Stratum	1	2	3	4	5	Group totals
Control	796	223	124	72	50	1265
Treated	104	104	105	103	105	521
Stratum totals	900	327	229	175	155	1786
Weight ATE	0.50	0.18	0.13	0.10	0.09	
Weight ATT	0.20	0.20	0.20	0.20	0.20	

Step 3 Pool stratum-specific treatment effects

Treatment Group	Stratum	Mean Percent Harsh Punishment	Standard Error
Untreated	1	0.097	0.009
Treated	1	0.114	0.028
Untreated	2	0.202	0.028
Treated	2	0.216	0.033
Untreated	3	0.198	0.027
Treated	3	0.195	0.030
Untreated	4	0.261	0.039
Treated	4	0.343	0.055
Untreated	5	0.336	0.068

$$\Delta = \sum_{k=1}^{K} w_k \Delta_k$$

$$SE(\Delta) = \sqrt{\sum_{k=1}^{K} w_k^2 \operatorname{var}(\Delta_k)}$$

```
    contrast      SE
ATE 0.014936 0.0205
ATT 0.013313 0.0250
```

Marginal Mean Weighting Through Stratification

- Marginal mean weighting through stratification (MMWS) consists of creating weights based on strata membership that adjust for the difference between the observed proportions of treated and untreated units within strata and the proportions that would be obtained if randomized treatment assignment was used.

- It is a similar idea to sampling weighting to adjust for disproprotionate stratified sampling.

Disproportionate stratified samples

- At least one stratum was over-sampled from the population, so there is a need for weights to adjust for the excess of data available form the over-sampled strata. The weight for each stratum is the inverse of the probability of selection of respondents in the stratum:

Example: Sample of high schools in Florida oversampling Title 1 Schools

Schools	Population	Sample	Sampling ratio	Weight
Title 1	1000	100	0.1	10
Not Title 1	4000	100	0.025	40

$$w_s = \frac{N_s}{n_s}$$

N_s: Population size of stratum s

n_s: Sample size of stratum s

Example

Proportion Treated	Proportion Untreated
0.29	0.71

Strata	Treated n_{s1}	Untreated n_{s0}	Total n_s
1	104	796	900
2	104	223	327
3	105	124	229
4	103	72	175
5	105	50	155

Marginal Mean Weigh Through Stratification for ATE

For units in propensity stratum *s* and assigned to condition *z*,

$$w_{sz} = \frac{n_s \times pr\left(Z = z\right)}{n_{sz}}$$

- where n_s is the number of units in stratum *s*,
- *Pr*(*Z* = *z*) is the marginal proportion receiving condition *z*
- $n_s \times pr(Z = z)$ is the *expected* number of units assigned to condition *z* in stratum *s* in a completely randomized experiment,
- n_{sz} is the *actual* number of units assigned to condition *z* in stratum *s*

Example Calculation of MMWS for ATE in Stratum 1

$$w_{sz} = \frac{n_s \times pr(Z = z)}{n_{sz}}$$

Number that should have been treated in stratum 1 if selection was random

$$w_{11} = \frac{900 \times 0.29}{104} = \frac{261}{104} = 2.51$$

Actual number treated in stratum 1

$$w_{10} = \frac{900 \times 0.71}{796} = \frac{639}{796} = 0.80$$

Marginal Mean Weigh Through Stratification for ATT

For units in propensity stratum *s* and assigned to condition *z*,

$$w_{sz} = Z + (1 - Z)\frac{n_{1s}\, pr(Z = 0)}{n_{0s}\, pr(Z = 1)}$$

- n_{1s} is the number of treated units in stratum *s*,
- N_{0s} is the number of untreated units in stratum *s*,
- $pr(Z = 1)$ is the marginal proportion treated
- $pr(Z = 0)$ is the marginal proportion untreated

Comparison of MMWS with IPTW for the ATT

$$w_i = Z_i + (1 - Z_i)\frac{\hat{e}_i}{1 - \hat{e}_i}$$

Odds of treatment

$$w_{sz} = Z + (1 - Z)\frac{n_{1s}\, pr(Z = 0)}{n_{0s}\, pr(Z = 1)}$$

Within-stratum non-parametric odds of treatment

Covariate Balance Evaluation with MMWS

- Covariate balance evaluation for MMWS can be performed in the same way as with IPTW.

- The balance of each covariate across treated and untreated groups is evaluated across strata rather than within each stratum.

- The *twang* package can be used to obtain standardized mean differences for all covariates

Example results of covariate balance evaluation

Effect	Minimum	Mean	Maximum	Unbalanced Covariates (Austin, 2011, Criteria)
ATT	–0.120	0.000	0.122	FR_SIZE, C0540
ATE	–0.125	0.006	0.188	C0114, C0158, C0268, C0272, C0274, FR_LVEL, FR_SIZE, C0540, C0542, C0556, IC0542, IC0546, IC0558

Comparisons Between IPTW & MMW-S

Under a correctly specified propensity score model

- IPTW adjustment removes 100% of bias
- MMW-S adjustment on the basis of five or six strata removes about 90% of bias

Robustness of MMW-S to Misspecified Propensity Score Model

When a nonlinear and/or non-additive propensity score model is misspecified as linear additive:

- IPTW adjustment typically leads to a bias proportional to the amount of confounding associated with the degree of nonlinearity
- The nonparametric procedure adopted by the MMW-S method is robust to propensity score misspecification when the misspecification does not change strata membership.

Estimation of Treatment Effect with MMWS

Horvitz and Thompson Estimator of the treatment effect (Rosenbaum, 1987)

$$\Delta = \frac{\sum_{i=1}^{n_1} w_{i1} y_{i1}}{\sum_{i=1}^{n_1} w_{i1}} - \frac{\sum_{i=1}^{n_0} w_{i0} y_{i0}}{\sum_{i=1}^{n_0} w_{i0}}$$

Estimation with weighted regression:

$$Y_i = \beta_0 + B_1 Z_i + e_i$$

Estimates of effect of one security personnel on proportion of harsh discipline

Effect	Estimate	Standard Error	p Value
ATE	0.019	0.023	.412
ATT	0.017	0.021	.411
Doubly-robust ATT	–0.024	0.061	.692

Doubly-robust estimation with propensity score stratification

- For estimation with within-strata means, regression estimation can be used to obtain the within-stratum means of treated and untreated groups.

- For estimation with weighted regression using the MMWS, the treatment effect estimate can be made doubly-robust by adding covariates to the model, and/or dummy stratum indicators.

Doubly robust estimation with weighted regression

$$Y_i = \beta_0 + B_1 Z_i + \sum_{s=1}^{4} \gamma_s S_i + \sum_{s=1}^{4} \lambda_s Z_i S_i + e_i$$

- When covariates and/or dummy stratum indicators are included in the outcome model, interactions between these variables and the treatment indicator should also be included.

- It is also necessary to center continuous covariates around the grand mean (for the ATE) or the treated group mean (for the ATT) to ensure that the estimate treatment effect can still be interpreted as the ATE or the ATT.

Summary of R packages useful for propensity score stratification

Package	Function	Objective
MatchIt	matchit, summary	Obtain strata based on propensity scores, and evaluate covariate balance
twang	bal.stat	Evaluate covariate balance
survey	svydesign, svymean, svyglm	Estimate weighted means, proportions, differences, and regression