Weighting Methods for Time-varying treatments

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Time-varying treatments

- Treatments that can be repeated at multiple occasions.

- The assignment of time-varying treatments depends on:
  - the previous treatment history
  - the previous outcomes
  - time-invariant covariates
  - time-varying covariates.

Example

- Estimate the effect of self-employment on job satisfaction from 1994 to 2012
- Data: National Longitudinal Survey of Youth 1979 (NLSY79).
- Measure of global job satisfaction: single question about one to five jobs per year,
- Measure of self-employment: Binary indicator measured in each survey wave.
- Sample size: n = 9150, waves = 10

IPTW for estimating the ATE of time-varying treaments

Selection bias can be controlled by weighting each observation at time *t* with the inverse of the probability of exposure to the conditions the individual was exposed to by time *t*.

$$w_{ti} = \frac{1}{\prod_{t=0}^{T} P(\mathrm{Z}_{ti} = z \mid \mathrm{Z}, \mathrm{W}, \mathrm{X})}$$

Time-invariant covariates

Treatment History

Time-varying covariates

Estimation of conditional probability of treatment at each wave

$$\text{logit}(Z_{ti} = 1 \mid \text{Z}, \text{W}, X) = \upsilon + \sum_{j=1}^{t-1} \beta_j Z_{ji} + \sum_{k=1}^{K} \gamma_k W_{ki} + \sum_{l=1}^{L} \pi_l X_{li}$$

- Estimation options:
  - One logistic regression model for each measurement wave.
  - A single model for all waves, accounting for clustering of observations within individuals (this can be a multilevel logistic regression model, a model with fixed effects of individuals, or a logistic regression model with correction for clustering).

The problem of extreme IPTW

0%	25%	50%	75%	100%
1.02e+00	1.13e+00	1.25e+00	1.53e+00	9.47e+08

- The occurrence of extreme weights is common in IPTW with repeatable treatments.
- Extreme weights result in biased estimates and large standard errors of the treatment effect.
- This problem can be solved by using stabilized weights

Stabilized IPTW

- It addresses the problem that the IPTW formula is likely to result in extreme weights.

- The numerator is the probability of treatment conditional of treatment history.

$$sw_{ti} = \prod_{t=1}^{T} \frac{P(\mathrm{Z}_{ti} = z \mid \mathrm{Z})}{P(\mathrm{Z}_{ti} = z \mid \mathrm{Z}, \mathrm{W}, \mathrm{X})}$$

Basic Stabilized IPTW

- It addresses the issue that the stabilized IPTW does not remove the confounding effect of treatment and requires that treatment history is included in the outcome model.

$$sw_{ti} = \prod_{t=1}^{T} \frac{P(Z_{ti} = z)}{P(Z_{ti} = z \mid Z, W, X)}$$

- The basic stabilized IPTW does not reduce extreme weights as much as the stabilized IPTW.

Stabilized and Basic Stabilized Weights from Example

- Stabilized weights:

0%	25%	50%	75%	100%
0.0101	0.9599	1.0020	1.0342	22.4729

- Basic stabilized weights:

0%	25%	50%	75%	100%
8.55e-08	4.61e-01	6.18e-01	7.85e-01	1.99e+03

Evaluation of covariate balance

- It can be done by examining standardized mean differences between treated and untreated groups across all treatment waves.

- The twang package can be used for covariate balance valuation.

- Covariate balance with stabilized weights:

```
Min.     1st Qu.  Median    Mean   3rd Qu.  Max.
-0.0659 -0.0104  0.0068  0.0101  0.0341  0.1250
Only one covariate had d > 0.1
```

Marginal Mean Weight Through Stratification

- This weight is less likely to have extreme values than the IPTW.
- It is obtained by creating strata at each measurement wave based on the probability of treatment assignment conditional on treatment history, time-varying and time-invariant covariates.
- Weights are obtained with:

$$w_{szt} = \frac{n_{st} P(Z_t = z)}{n_{szt}}$$

Estimating the treatment effects

- Marginal models can be used to estimate the mean of all individuals with the same values of the covariates:
  - Weighted regression estimation with cluster-robust standard errors
  - Generalized estimating equations

- With marginal models, the correlations between observations obtained from the same individual are considered nuisance parameters.

Weighted regression estimation

Number of previous treatments

Measurement wave coded as 0 to 9

$$y_{ti} = \beta_0 + \beta_1 Z_{ti} + \beta_2 Z_{t-1i} + \beta_3 W_{ti} + \beta_4 T_{ti} + \beta_4 Z_{ti} Z_{t-1i} + \beta_5 Z_{ti} W_{ti} + \beta_4 Z_{ti} T_{ti} + \varepsilon_i$$

Current treatment

Time-varying covariate

- Adjustment of standard errors for clustering was obtained with Taylor-series linearization, but resampling methods such as the bootstrap could also be used.

Results with weighted regression

Model in R: genSatis ~ selfEmploy + cumSE + experience + timeRecoded + selfEmploy:cumSE + selfEmploy:experience + selfEmploy:timeRecoded

```
 Estimate Std. Error t value Pr(>|t|)
(Intercept)            3.28311    0.00725  453.14  < 2e-16 ***
selfEmploy             0.19372    0.01879   10.31  < 2e-16 ***
cumSE                  0.02181    0.00553    3.94  8.1e-05 ***
experience             0.04518    0.00403   11.22  < 2e-16 ***
timeRecoded            0.00616    0.00111    5.53  3.3e-08 ***
selfEmploy:cumSE       0.00618    0.00817    0.76    0.449
selfEmploy:experience -0.02596    0.01151   -2.26    0.024 *
selfEmploy:timeRecoded -0.02490   0.00438   -5.68  1.4e-08 ***
```

Conclusion: self-employed individuals have higher global satisfaction than individuals not self-employed, they have a slower rate of increase in satisfaction over time and over their tenure in their position than individuals not self-employed.

Generalized Estimating Equations (GEE)

Function for means:

$$\mu_{ti} = E(Y_{it} \mid Z_{ti}, Z_{t-1i}, W_{ti})$$

$$g(\mu_{ti}) = \beta_0 + \beta_1 Z_{ti} + \beta_2 Z_{t-1i} + \beta_3 W_{ti} + \beta_4 T_{ti} + \beta_4 Z_{ti} Z_{t-1i} + \beta_5 Z_{ti} W_{ti} + \beta_4 Z_{ti} T_{ti} + \varepsilon_i$$

Variance Function:

$$V(Y_{ti}) = \mathrm{var}(Y_{ti} \mid Z_{ti}, Z_{t-1i}, W_{ti})$$

Correlation structure:

$$cor(Y_{it2}, Y_{it1}) = \alpha^{t2-t1}$$

Results with GEE

	Estimate	Std.err	Wald	Pr(>|W|)	
(Intercept)	3.28057	0.00739	1.97e+05	< 2e-16	***
selfEmploy	0.12342	0.02044	3.65e+01	1.6e-09	***
cumSE	0.01767	0.00533	1.10e+01	0.00092	***
experience	0.01179	0.00369	1.02e+01	0.00140	**
timeRecoded	0.00685	0.00116	3.50e+01	3.4e-09	***
selfEmploy:cumSE	0.01383	0.00782	3.13e+00	0.07696	.
selfEmploy:experience	-0.02720	0.01435	3.59e+00	0.05800	.
selfEmploy:timeRecoded	-0.01799	0.00457	1.55e+01	8.3e-05	***

R packages

Package	Function	Objective
ipw	ipwtm	Estimate IPTW, SIPTW, and BSIPTW
survey	svyglm	Treatment effect estimation with regression with cluster-robust standard errors
geepack	geeglm	Treatment effect estimation with generalized estimating equations